Optimization in Networked Systems

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RESEARCH MAP

Scale

High

Socio physics (Complex networks)

Complex adaptive systems

Multi-agent systems

Game theory

Collective systems

Low

Self-interest seeking Adaptability

Low

High
A large collection of people are smarter than an elite few.

In “the Wisdom of Crowds”, Surowiecki,(2004) suggests new insights regarding how our social and economic activities should be organized.

The wisdom of crowds emerges only under the right conditions (diversity, independence, etc)
Phase Transition in Collective Behavior

- Crowds are wise, but are also often foolish.

Then under what mechanism can we improve the performance of collective systems?

The way of interaction, the network topology, plays a crucial role.
Emergence by Nature

- Emergence by nature (empirical view)
- View emergence as an “innate property” of natural systems
  “Systems self-organize into a complex state, poised between predictable cyclic behavior and unpredictable chaos”
- Inspires research to discover and explain emergent behaviors
Emergence by Design

- Emergence by design strategies (operational view)
  “System-wide behavior emerges from interactions among individual elements”
  - Some researchers view emergence as a property that is “designed” into systems
  - Inspire research into design techniques to induce desired emergent behaviors
Emergence by Design:Illusion of Control?

Internet throughput

self-similarities

phase transitions

Meta-stabilities
distribution of call types in wireless cells

difficult to predict and control because of phase-transitional behavior

unordered equilibrium chaos (self-organized criticality)

turbulence oscillation

congestion collapse

high load and the network topology play crucial roles
Emergence by Design: Evolutionary Optimization (1)

- **Design of Communication Networks**
  Tradeoff between congestion and network design cost

- **Diffusion of Innovation**

- **Consensus (synchronization) in Networked Systems**
A Network Flow Model

- Packet generation
  - Packet is generated at random with some rates
- Each nodes process one packet per time
- Each node has a queue to store undelivered packets
- Routing: Shortest path
- Traffic congestion is determined by node betweenness
  : total shortest paths through the node
Optimized Networks:
Minimizing congestion

- Network size: 32 nodes
- Fixed number of links

**Optimal network**
- Star network: packet generation rate is small
- Random network: packet generation rate is high

\[
\beta_i \in \theta^* (\rho)
\]

\[
\langle \beta \rangle: \text{Average node betweenness}
\]

\[
\theta = \max_{i \in N} \beta_i - \langle \beta \rangle / \langle \beta \rangle
\]

Optimization in complex network, (Ferrere, 2006)
Adjacency Matrix of Graph

- The cording of the adjacency matrix: \( A = (a_{ij}) \)
  - Node \( i \) and node \( j \) is connected: \( a_{ij} = 1 \)
  - Node \( i \) and node \( j \) is not connected: \( a_{ij} = 0 \)

Adjacency matrix

\[
A = \begin{pmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{pmatrix}
\]

Example:
Stochastic Optimization

Simulated annealing
- Probabilistic algorithm for the optimization problem
- Rewiring trials - Rewiring a randomly selected link
- Fitness function to be optimized: $Q$
  - if $\delta Q = Q_{\text{final}} - Q_{\text{initial}} < 0$
    accept rewiring

Optimized network
- $N = 50$, and $\langle k \rangle = 4$
The Fitness Function (1)
Link Density

- Design cost: the link density $\alpha$

\[ \alpha = \frac{1}{n \binom{n}{2}} \sum_{i<j} a_{ij} \]

- Maximum possible links of the network with $n$ nodes: $n \binom{n}{2}$
- The number of links $\sum_{i<j} a_{ij}$

\[
\begin{array}{c|cccc|c|c|c|c|c|c|c|c|}
\hline
& 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 1 & 1 & 0 \\
2 & 1 & 0 & 0 & 1 & 0 \\
3 & 1 & 0 & 0 & 0 & 1 \\
4 & 1 & 1 & 0 & 0 & 0 \\
5 & 0 & 0 & 1 & 0 & 0 \\
\hline
\end{array}
\]

$\sum_{i<j} a_{ij} = 5$
The Fitness Function (2)  
Congestion Index

- **Congestion measure**: $\lambda (\rho)$
  - Packet generation probability on certain node: $\frac{\rho}{n-1}$  
    $\rho$: packet generation rate
  - Quantity of packet input on k node: $\frac{\rho}{n-1} \times \beta_k$
  - Quantity of packet output: $\beta_k$: betweenness at k node
  - Queue length average on k node: $\frac{\rho \beta_k}{1 - \rho \frac{\beta_k}{n-1}}$  
    Little’s law

Congestion measure
- Total queue length on the network:

$$\lambda (\rho) = \sum_{k \in N} \frac{\rho \beta_k}{n-1} \frac{1 - \rho \beta_k}{n-1}$$
The Weighted Fitness Function

- **Link density**: $\alpha$
- **Congestion function**: $\lambda(\rho)$
  - $\rho$: Packet generation rate
- **Weight**: $\omega \quad 0 \leq \omega \leq 1$

The weighted object function to be minimized: $E(\omega, \rho)$

$$E(\omega, \rho) = \omega \lambda(\rho) + (1 - \omega)\alpha$$

$$\lambda(\rho) = \sum_{k \in N} \frac{\rho}{1 - \rho} \frac{\beta_k^i}{n - 1}$$

$$\alpha = \frac{1}{n C_2} \sum_{i<j} a_{ij}$$
Generic Algorithm

**MGG Model (Minimal Generation Gap)**

- **Crossover rate**: 0.7
- **Mutation rate**: $2/nC_2$

**Generic code representation**

**Number of Individuals**: 10 (networks)

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Fig. 1. The basic scheme of the minimization algorithm.
An Initial Network

**Initial**: Random network
- **A fixed number of nodes**: 100
- **Links creation**
  - Poisson distribution
  - 7 link per each node

![The degree distribution](image)

The degree distribution
Optimized Network (1)

\[ E(\omega, \rho) = \omega \lambda(\rho) + (1 - \omega) \alpha \]

\( \omega = 1 \): Optimizing only congestion function
(packet generation rate: \( \rho = 0.3 \))

- Optimal network: Complete network
  - Average link per node: 99.9
  - Link density: 0.9999 (4949/4950)
  - Congestion function value: 0 → no congestion

The degree distribution
Optimized Network (2)

\[ E(\omega, \rho) = \omega \lambda(\rho) + (1 - \omega)\alpha \]

- \( \omega = 0 \): Minimizing only link density
  - (packet generation rate: \( \rho = 0.3 \))
  - Optimal network: Tree-like network
    - Average link: 1.98
    - Link density: 0.02 (99/4950)
    - Congestion index: 0.027
  - Tree structure has the smallest links
Optimized Networks (3)

\[ E(\omega, \rho) = \omega \lambda(\rho) + (1-\omega)\alpha \quad 0 \leq \omega \leq 1 \]

\[ \omega \]

\[ \rho \]

\[ \alpha \]

\[ \lambda \]

\[ \lambda(\rho) \]

\[ \lambda \]

\[ \omega \lambda(\rho) \]

\[ (1-\omega)\alpha \]

\[ E(\omega, \rho) \]

\[ 0 \leq \omega \leq 1 \]

\[ 0 \leq \rho \leq 1 \]

\[ \lambda \]

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Summary:
Optimal Traffic Networks

Phase 1: Tree $\rightarrow$ Hub $\rightarrow$ Star
: The link density increases slowly $\alpha \approx 0.02 \rightarrow 0.15$

Phase 2: Star $\rightarrow$ Random $\rightarrow$ Complete
: Link density increases suddenly $\alpha = 0.15 \rightarrow \alpha \approx 1$
Emergence by Design: Evolutionary Optimization (2)

- Design of Communication Networks
  Tradeoff between congestion and design cost

- Diffusion of Innovation

- Consensus (synchronization) in Networked Systems
Diffusion of Innovation

- Why the markets occasionally accept innovations rather slowly compared with the superior technological advances of the innovation?
  “The slow pace of the fast change” (B. Chakravorti, 2003)

Installed base of facsimile machine in North America (Rohlfs)
Concept of diffusion and contagion arises in many fields

- Spread of infectious disease
- Diffusion of innovations
- Emergence of uncertainty in economic beliefs
- Transmission of cultural fads

Question 1: In what sense are these phenomena the same and how are they different?

Question 2: What conditions trigger the decision to adopt something?
An Epidemic Diffusion Model (1)

The SIR model

- Consider a fixed population of size $N$
- Each individual is in one of three states:
  - Susceptible (S), Infected (I), Removed (R)

$$S \xrightarrow{\beta} I \xrightarrow{\lambda} R$$

- Dynamic process: Mixing model
At each time step, each individual comes into contact with another individual chosen uniformly at random
An Epidemic Diffusion Model (2)

- Each node may be in the following states
  - Susceptible (S) (unaware, also inactive, non-adopter)
  - Infected (I) (aware, also active, informed, adopter)
  - Removed (R) (lose interest or forget)

- Infection rate $\beta$: probability of getting infected by a neighbor per unit time
- Immunization rate $\gamma$: probability of a node getting recovered per unit time
Universal Property of Diffusion Models

• Global Infection only occur after a threshold (critical mass)

• Many models on epidemic spreads, information cascades, fads, have the same threshold property

• Susceptible become infected through their contacts with infected individuals at a rate $\beta$

• Infected agents are removed at rate $\gamma$

• There is a threshold above which the diseases spread through the population

$$\frac{\beta}{\gamma} = \lambda_c$$

$\lambda_c$ Critical mass: threshold property in social dynamics

• The network topology affects critical mass
The critical mass is given at

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle}$$

$$\langle k \rangle$$: average degree of node

Scale-free network does not have critical mass
Social Influence Networks

Peer influence creates consensus within small social groups

Impact of opinion leaders may be large

Local networks

Scale-free networks

hub agent
Dominant Eigenvalue of Adjacency Matrix

Example: Adjacency matrix: symmetric

\[
A = \begin{pmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Eigenvalue of symmetric matrix \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \)

\[ \lambda_1 \left( A \right) \] is the largest eigenvalue of the adjacency matrix \( A \)

\[ \lambda_c = \frac{\beta}{\gamma} = 1 / \lambda_1 \left( A \right) \] is the epidemic threshold.
Convergence of Diffusion Process

- The expected state of the system at time $t$ is given by

$$v^t = (pA + (1-q)I)v^{t-1}$$

- As $t \to \infty$
  - if $\lambda_1(pA + (1-q)I) < 1 \Leftrightarrow \lambda_1(A) < \gamma/\beta$ then $v^t \to 0$
  - the probability that all copies die converges to 1
  - if $\lambda_1(pA + (1-q)I) = 1 \Leftrightarrow \lambda_1(A) = \gamma/\beta$ then $v^t \to c$
  - the probability that all copies die converges to 1
  - if $\lambda_1(pA + (1-q)I) > 1 \Leftrightarrow \lambda_1(A) = \gamma/\beta$ then $v^t \to \infty$
  - the probability that all copies die converges to a constant $< 1$

$\lambda_1(A)$ is the largest eigenvalue of the adjacency matrix $A$.
The Largest Eigenvalue

\[ \lambda_{\text{max}} = \max \frac{x^\top Ax}{\|x\|^2} \]

\[ d_{\text{average}} \leq \lambda_{\text{max}} \leq d_{\text{max}} \]

\[ \sqrt{d_{\text{max}}} \leq \lambda_{\text{max}} \leq d_{\text{max}} \]

If \( G \) is regular of degree \( d \), then \( \lambda_{\text{max}} = d \).
An Eigenvalue Point of View

\[ \lambda_1(A) : \text{the largest eigenvalue of the adjacency matrix } A \]

- **Object 1:** Minimizing spread of diffusion
- **Object 2:** Maximizing spread of diffusion

\[ \lambda_1 = pN = \langle k \rangle \quad \lambda_1 = \sqrt{N - 1} \]

\[ \lambda_1 \simeq N^{1/4} \quad \lambda_1 = N - 1 \]

\( p: \text{connection probability} \)
Minimizing Diffusion

- **Object function 1:** \[ \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \]

Minimize the largest eigenvalue

\[ F = \omega \lambda_1 + (1 - \omega) <k> \]

\(<k>: \text{average degree}\)

\(\omega = 0.1\)
\[ \lambda_1 = 2.38 \quad \lambda_n = -2.38 \quad <k> = 1.98 \]

\(\omega = 0.5\)
\[ \lambda_1 = 2.38 \quad \lambda_n = -2.38 \quad <k> = 1.98 \]

\(\omega = 0.9\)
\[ \lambda_1 = 2.48 \quad \lambda_n = -2.48 \quad <k> = 1.98 \]

**Line structure is optimal for minimizing diffusion**
Maximizing Diffusion

- Object function 2:

Maximize the largest eigenvalue

\[
F = \omega / \lambda_1 + (1 - \omega) \rho
\]

\(\rho\) link density

\(\rho = <k> / (N - 1)\)

\(<k>\): average degree

Star networks

<table>
<thead>
<tr>
<th>(\omega)</th>
<th>(\lambda_1)</th>
<th>(\lambda_n)</th>
<th>(&lt;k&gt;)</th>
</tr>
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<tbody>
<tr>
<td>0.1</td>
<td>6.42</td>
<td>-3.10</td>
<td>2.28</td>
</tr>
<tr>
<td>0.2</td>
<td>8.96</td>
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<td>2.66</td>
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<td>0.3</td>
<td>10.23</td>
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<tr>
<td>0.4</td>
<td>13.01</td>
<td>-3.10</td>
<td>3.52</td>
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<td>0.5</td>
<td>13.88</td>
<td>-3.28</td>
<td>3.76</td>
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<tr>
<td>0.6</td>
<td>17.04</td>
<td>-3.08</td>
<td>4.68</td>
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<tr>
<td>0.7</td>
<td>19.99</td>
<td>-3.13</td>
<td>5.76</td>
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<td>24.08</td>
<td>-2.79</td>
<td>7.5</td>
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<tr>
<td>0.9</td>
<td>29.87</td>
<td>-3.02</td>
<td>10.62</td>
</tr>
<tr>
<td>1</td>
<td>100.0</td>
<td>-1.98</td>
<td>98.9</td>
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</table>

Complete networks

<table>
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<tr>
<th>(\omega)</th>
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Core dense network is optimal for maximizing diffusion
Emergence by Design: Evolutionary Optimization (3)

- **Design of Communication Networks**
  Tradeoff between congestion and design cost

- **Diffusion of Innovation**

- **Consensus (synchronization) in Networked Systems**
Consensus Problems

"Consensus" means to reach an agreement regarding a certain quantity of interest that depends on the state of all nodes (subsystems).

More specific, a consensus algorithm is a decentralized rule that results in the convergence of the states of all network nodes to a common value.

\[ x_i = x_j = \ldots = x_{\text{consensus}} \]
Consensus Problems in Engineering

A position reached by a group as a whole

Battle space management scenario illustrating distributed command and control between heterogeneous air and ground assets
Synchronization: Prevalent appearance in physics and biology

Homogeneity is important for better synchronization
“Consensus has connections to problem in synchronization”

Model: every bird adjusts its velocity by adding to it a weighted average of the differences of its velocity with those of the other birds. That is, at time $t \in \mathbb{N}$, and for bird $i$,

$$v_i(t + 1) - v_i(t) = \sum_{j=1}^{k} a_{ij}(v_j(t) - v_i(t)).$$

“Emergent behavior on flocks”

Engineering Problems

Question: How do we add some new links with better consensus?
Observation:

No matter how large the network is, a globally coupled network will synchronize if its coupling strength is sufficiently strong.

Good – if synchronization is useful.
Observation:
No matter how strong the coupling strength is, a locally coupled network will not synchronize if its size is sufficiently large.

Good - if synchronization is harmful
Synchronization in Small-World Networks

Start from a nearest neighbor coupled network

Add a link, with probability $p$, between a pair of nodes

Good news: A small-world network is easy to synchronize!

Connectivity of networks does matter for synchronization

Laplacian matrix = Degree – Adjacency matrix

$\lambda_1 = 0$ is always an eigenvalue of a Laplacian matrix

$0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n \leq 2\Delta$

$\Delta = \max_i d_i$

$\lambda_n / \lambda_2 : \text{algebraic connectivity}$

$: \text{Smaller algebraic connectivity}$

$: \text{better consensus formation}$
The distributed consensus algorithm

\[ x_i(t+1) = x_i(t) + \varepsilon \sum_{i \in N_i} w_{ij} (x_j(t) - x_i(t)) \]

Convergence to the average of the initial values of all agents

\[ x_1 = x_2 = \ldots = x_n = \sum_i x_i(0) / n \]

The weighted adjacency matrix \( G = (w_{ij}) \)

(i) Graph \( G \) is connected
(ii) \( G \) is balanced: symmetric graph

\[ \sum_{i \neq j} w_{ij} = \sum_{j \neq i} w_{ji} \]
Convergence in Consensus Problems

- circle
  - average link: 2
  - $\lambda_n / \lambda_2 = 365$
    - $\lambda_2 : 0.01$
    - $\lambda_n : 4$

- line
  - average link: 2
  - $\lambda_n / \lambda_2 = 1458$
    - $\lambda_2 : 0.003$
    - $\lambda_n : 4$

- complete network
  - average link: 60
  - $\lambda_n / \lambda_2 = 1$
    - $\lambda_2 : 60$
    - $\lambda_n : 60$

Initial value of each agent: $x_i(0) = i$
$E(\omega) = \omega \cdot \frac{\lambda_n}{\lambda_2} + (1 - \omega) \cdot \alpha$

**Ramanujam network**

**Optimized Networks (1)**

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<thead>
<tr>
<th>$\omega$</th>
<th>$\lambda_2$</th>
<th>$\lambda_n$</th>
<th>$\lambda_n/\lambda_2$</th>
<th>$\alpha$</th>
<th>$P$</th>
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<tr>
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<tr>
<td>0.3</td>
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<td>8.64</td>
<td>5.25</td>
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<tr>
<td>0.4</td>
<td>2.16</td>
<td>9.49</td>
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<td>0.5</td>
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<td>10.69</td>
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**Random networks**

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Optimized Networks (2)

\[ E(\omega) = \omega \cdot \frac{\lambda_n}{\lambda_2} + (1 - \omega) \cdot \alpha \]

\[ \omega=0.6 \]
\[ \lambda_2=3.4 \quad \alpha=6.9 \quad P=0.47 \]
\[ \lambda_n=11.46 \]
\[ \lambda_n/\lambda_2=3.35 \]

\[ \omega=0.7 \]
\[ \lambda_2=4.64 \quad \alpha=8.3 \quad P=0.51 \]
\[ \lambda_n=13.35 \]
\[ \lambda_n/\lambda_2=2.87 \]

\[ \omega=0.8 \]
\[ \lambda_2=6.17 \quad \alpha=10.2 \quad P=0.50 \]
\[ \lambda_n=15.63 \]
\[ \lambda_n/\lambda_2=2.53 \]

\[ \omega=0.9 \]
\[ \lambda_2=8.34 \quad \alpha=12.8 \quad P=0.31 \]
\[ \lambda_n=18.57 \]
\[ \lambda_n/\lambda_2=2.23 \]

Random network

\[ \lambda_2=1.58 \quad \alpha=6.9 \quad P=1.37 \]
\[ \lambda_n=13.89 \]
\[ \lambda_n/\lambda_2=8.75 \]

\[ \lambda_2=2.08 \quad \alpha=8.3 \quad P=1.99 \]
\[ \lambda_n=15.86 \]
\[ \lambda_n/\lambda_2=7.6 \]

\[ \lambda_2=3.87 \quad \alpha=10.2 \quad P=1.77 \]
\[ \lambda_n=19.15 \]
\[ \lambda_n/\lambda_2=4.95 \]

\[ \lambda_2=5.53 \quad \alpha=12.4 \quad P=0.95 \]
\[ \lambda_n=21.02 \]
\[ \lambda_n/\lambda_2=3.8 \]
Comparison of Convergence Speed

- Initial value of each agent: $x_i(0)=i$

- **Optimal network**
  - average link = 5
  - average link = 11

- **Random network**
  - average link = 5
  - average link = 11
Multi-graph Topologies

- Minimizing spread
- Minimize the spread of cascade failure or infective diseases

- Maximizing spread
- Maximize the influence in voting campaign

Synchronization,
Maximize the effect of coordinated behavior
Five Stages of Research

1) **Observe:** Gather data to demonstrate power law behavior in a system.
2) **Interpret:** Explain the import of this observation in the system context.
3) **Model:** Propose an underlying model for the observed behavior of the system.
4) **Validate:** Find data to validate (and if necessary specialize or modify) the model.
5) **Design (Control):** Design ways to control and modify the underlying behavior of the system based on the model.

Lots of open research problems in the design of complex systems
Conclusion

- Social systems involve a large-scale self-interested individual decisions that are main obstacles as well as driven forces for improving social systems.

- Social improvements that requires persuasion and consensus among us become very slow since most social influence networks are asymmetric.

- Evolutionary optimization is a powerful method for designing desirable social systems.
Thank you for listening!!

Question Time